

Phase field simulations for grain growth in materials containing second-phase particles

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Research group: Thermodynamics in materials engineering

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- **Introduction on Zener pinning**
- **Phase field model**
- **Grain boundary/particle interactions**
- **Large-scale 2D-simulations**
- **Conclusions**

- **Grain growth:**

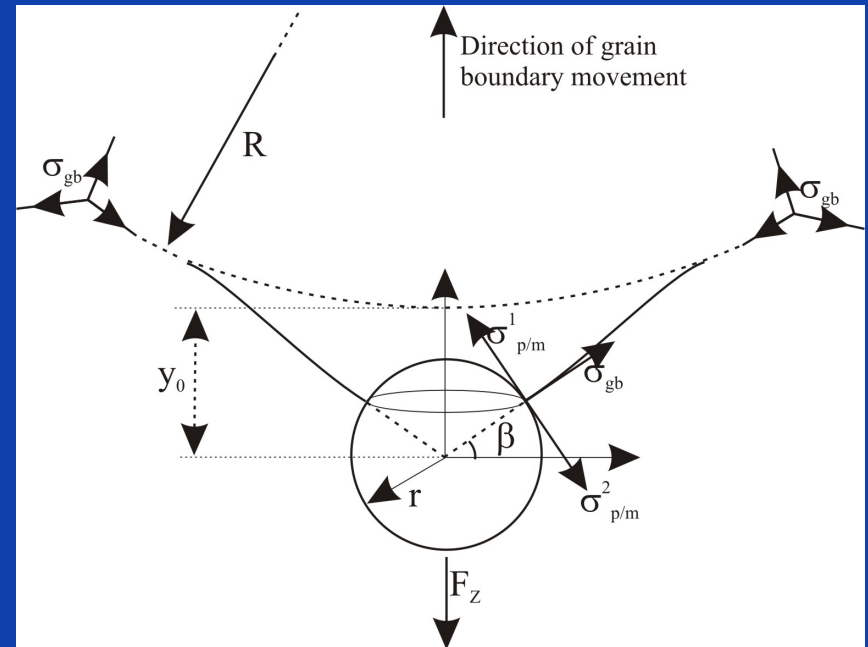
$$P = \sigma_{gb} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

- **Zener pinning:**

$$F_Z^{\max} = \pi r \sigma_{gb} (3D)$$

- **'Dimple shape'**
- **Limiting grain size:**

$$\frac{\overline{R}_{\lim}}{r} = \frac{b}{f_V^\beta}$$



- **Extension of model of D. Fan and L.-Q. Chen for normal grain growth**

- **Phase field variables:**

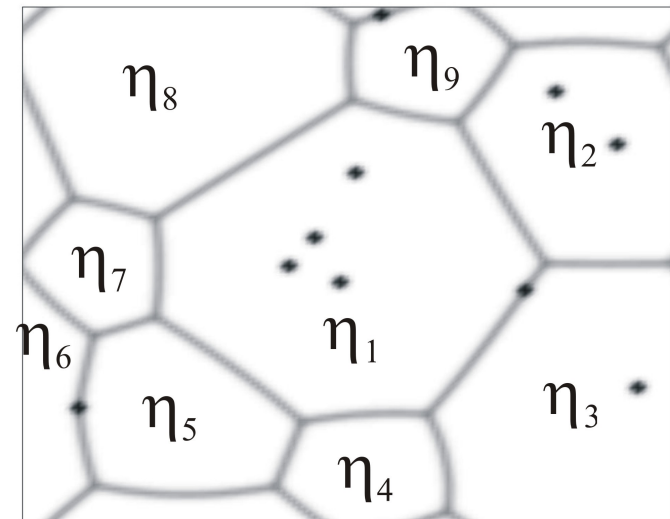
$$\eta_1, \eta_2, \dots, \eta_i, \dots, \eta_p$$

- **Particles : $\Phi=1$**

$$(\eta_1, \eta_2, \dots, \eta_i, \dots, \eta_p) = (0, 0, \dots, 0, \dots, 0)$$

- **Grain i of matrix-phase : $\Phi=0$**

$$(\eta_1, \eta_2, \dots, \eta_i, \dots, \eta_p) = (0, 0, \dots, 1, \dots, 0)$$



- **Free energy**

$$F = \int_V \left[\sum_{i=1}^p \left(\frac{\alpha}{4} \eta_i^4 - \frac{\beta}{2} \eta_i^2 \right) + \gamma \sum_{i=1}^p \sum_{j \neq i}^p \eta_i^2 \eta_j^2 + \epsilon \Phi \sum_{i=1}^p \eta_i^2 + \sum_{i=1}^p \frac{\kappa_i}{2} (\nabla \eta_i)^2 \right] dV$$

- **Equilibrium**

- $\Phi=0$: $(\eta_1, \eta_2, \dots, \eta_p) = (1, 0, \dots, 0), (0, 1, \dots, 0), \dots, (0, 0, \dots, 1), (-1, 0, \dots, 0), \dots$
- $\Phi=1$: $(\eta_1, \eta_2, \dots, \eta_p) = (0, 0, \dots, 0)$

- **Kinetic equations (Ginzburg-Landau)**

$$\frac{\partial \eta_i(\vec{r}, t)}{\partial t} = -L_i \frac{\partial F}{\partial \eta_i(\vec{r}, t)} = -L_i \left(\frac{\partial f(\eta_1, \eta_2, \dots)}{\partial \eta_i(\vec{r}, t)} - \kappa_i \nabla^2 \eta_i(\vec{r}, t) \right)$$

- **Equations**
$$\frac{\partial \eta_i(\vec{r}, t)}{\partial t} = -L_i \left(\frac{\partial f(\eta_1, \eta_2, \dots)}{\partial \eta_i(\vec{r}, t)} - \kappa_i \nabla^2 \eta_i(\vec{r}, t) \right)$$
- **with**
$$f = m \left(\sum_{i=1}^p \left(\frac{\eta_i^4}{4} - \frac{\eta_i^2}{2} \right) + \sum_{i=1}^p \sum_{j \neq i}^p \eta_i^2 \eta_j^2 \right) + \varepsilon \Phi \sum_{i=1}^p \eta_i^2$$
- **Grain boundary energy (expressed in J/g.p.)**

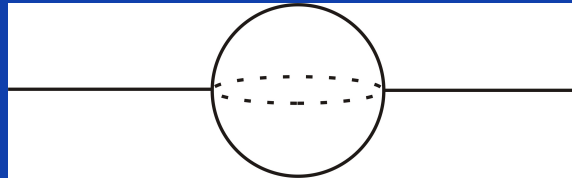
$$\kappa = 0.5, m = 1 \Rightarrow \sigma_{gb} = 0.40$$

$$\kappa = 4, m = 1 \Rightarrow \sigma_{gb} = 1.16$$

$$\kappa = 1, m = 0.25 \Rightarrow \sigma_{gb} = 0.29$$
- **Interfacial energy particles:** $m / 2 < \varepsilon < 2.5$
- **Interfacial thickness :** $\propto \sqrt{\frac{\kappa}{m}}$
- **Growth rate :** $\propto \kappa L \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$

- Energetic consideration:

- Geometry:



- Interaction energy:

$$2r\sigma_{gb} \quad (2D)$$

$$\pi r^2 \sigma_{gb} \quad (3D)$$

- Diffuse grain boundaries \Rightarrow

- Interaction energy slightly too negative
 - Lower limit on particle size

- Evolution spherical grain boundary pinned by single particle

- 'dimple shape'

- **Isotropic grain boundary properties:**

$$\kappa_i = 0.5, L_i = 1, m = 1, \varepsilon = 1$$

- **Round particles:** $r = 2.5$

$$r = 3$$

- **Area fraction:**

$$f_a = 0.004 - 0.16$$

- **System size: 256x256 and 512x512**

- **Initial microstructure:**

- Grain nucleation in presence of particles ($R_0=0$)
- Grain nucleation and initial grain growth without particles ($R_0>0$)

- **Random particle distribution**

- High scatter for low f_a

- **Fitting:**
$$\frac{\overline{R_{\text{lim}}}}{r} = \frac{b}{f_a^\beta}$$

- **Theory:**

$$\beta = 0.5$$

- **Phase field ($R_0=0$):**

$$\beta = 0.46, b = 1.38$$

- **Monte Carlo:**

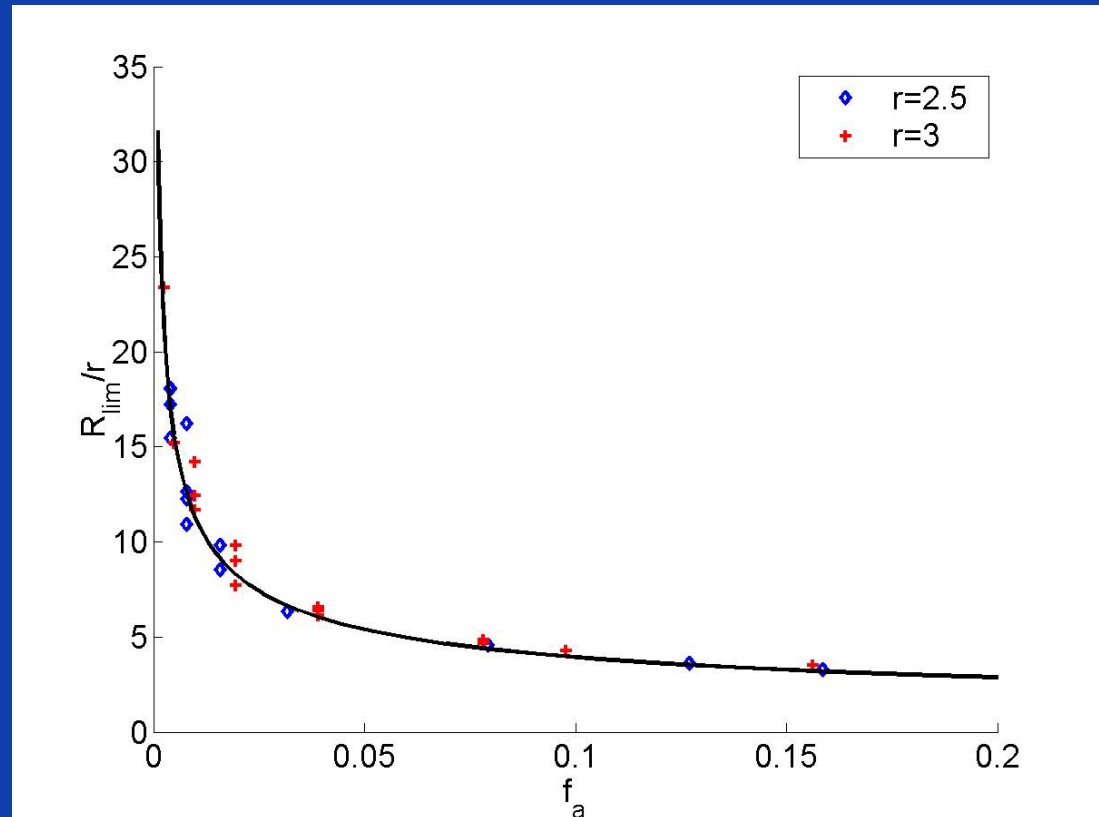
$$\beta = 0.5, b = 1.7$$

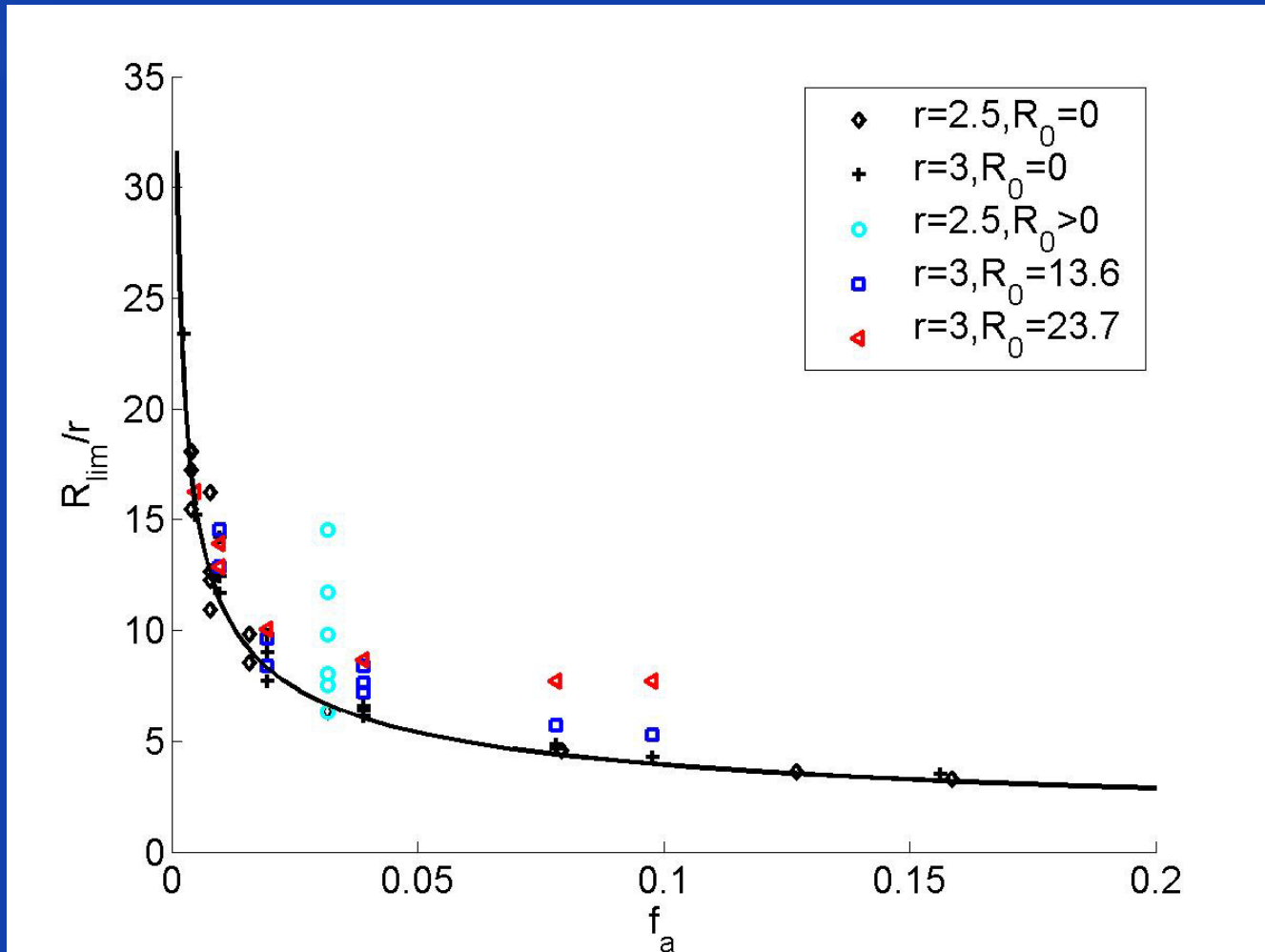
$$\beta = 0.54, b = 1.2$$

- **Front-tracking:**

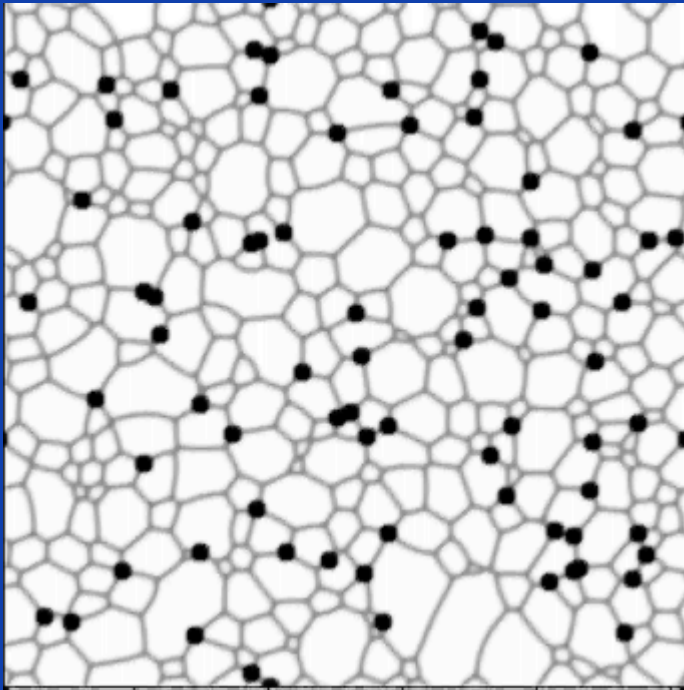
$$\beta = 0.46$$

$$\beta = 0.5$$



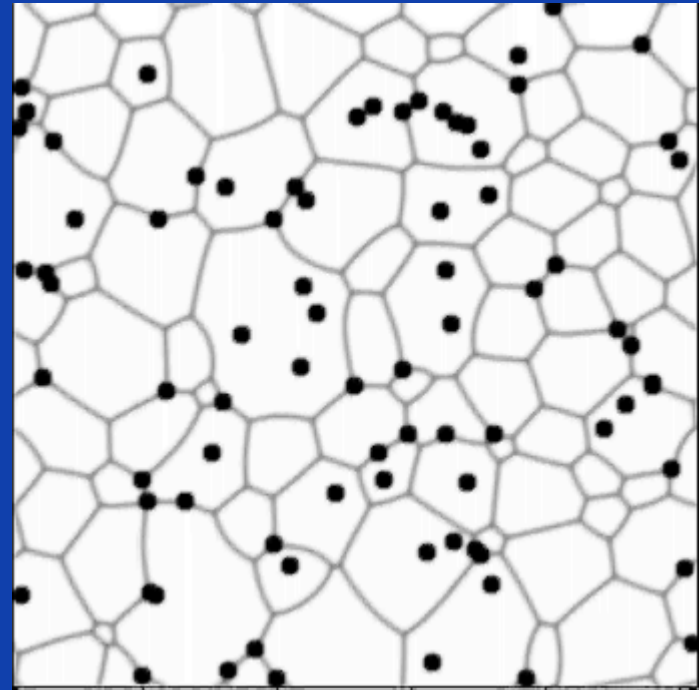


- $R_0 = 0$:



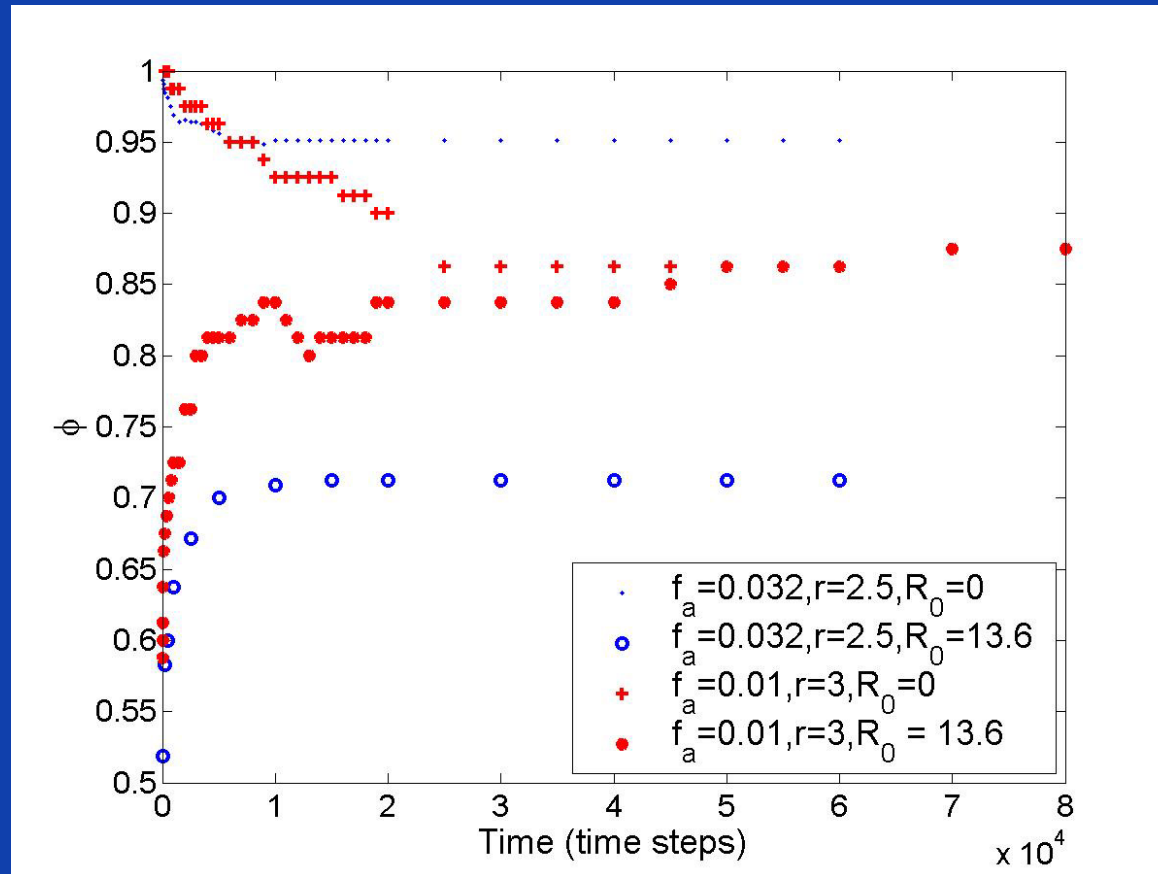
$$r = 3, f_a = 0.04, \bar{R}_0 = 0$$

- $R_0 > 0$

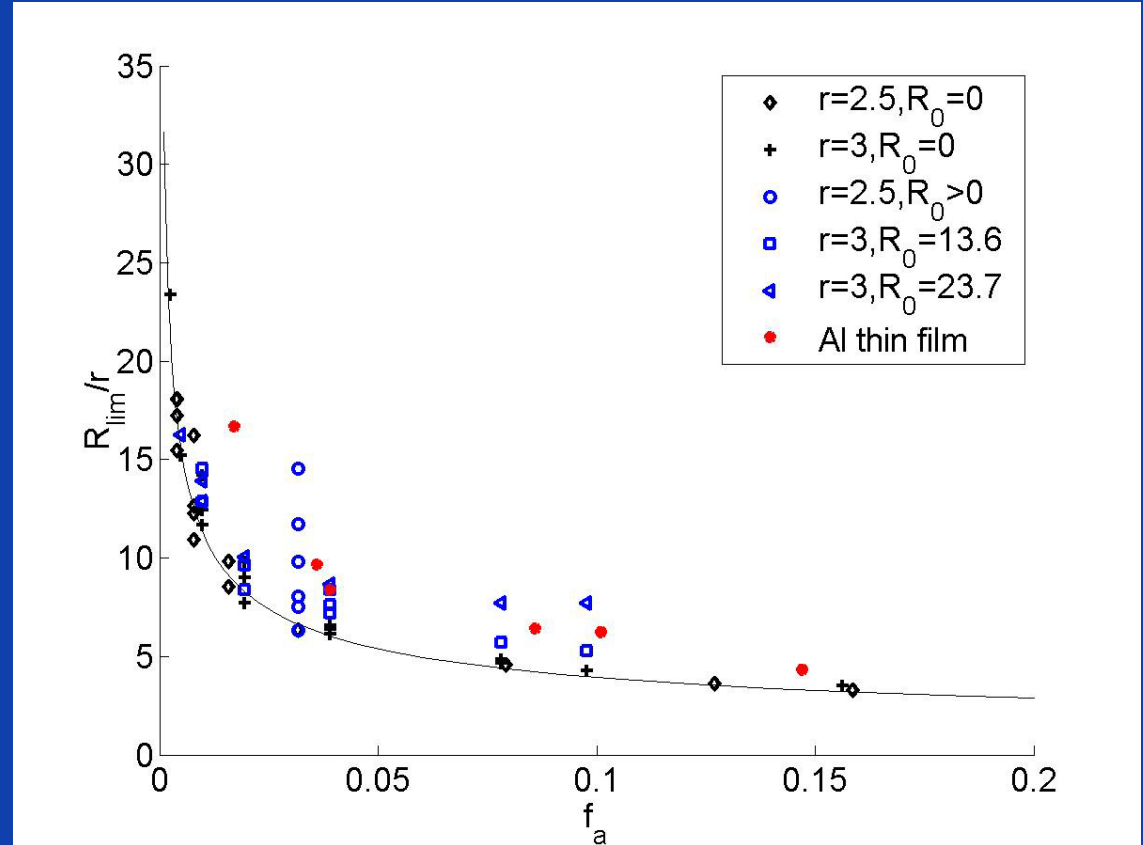


$$r = 3, f_a = 0.04, \bar{R}_0 = 13.6$$

- Fraction of particles on grain boundaries: temporal evolution



- Thin Al-films
 - Columnar grains
 - 2D- structure
 - CuAl_2 -particles



Data from H.P. Longworth and C.V. Thompson

- Zener pinning was studied as a function of
 - Particles size
 - Number of particles
 - Initial microstructure
- Long computation times even for simple materials
- Further study of ‘dimple shape’ and dynamic behavior of the grain boundaries is necessary

- **N. Moelans, B. Blanpain, P. Wollants, "A phase field model for the simulation of grain growth in materials containing finely dispersed incoherent second-phase particles", *Acta Mater.*, 2005, in press.**
- **N. Moelans, B. Blanpain, P. Wollants, "Phase field simulations of grain growth in 2-dimensional polycrystalline systems containing finely dispersed incoherent second-phase particles", in preparation.**
- **Available on <http://www.ulyssis.org/~nele/>**

